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Lattice Calculations of Glueball Properties*

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Abstract

The physical and theoretical reasons for being interested in lattice calculations of glueball properties are briefly reviewed. Some desirable traits of that non-experts can look for in numerical calculations are pointed out. The status of glueball spectrum calculations in lattice gauge theory is summarized by presenting a plot of dependence of mass ratios on the finite size of the box.

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1 Introduction

Almost everyone expects the force that confines quarks to confine gluons as well. However, the resulting bound states—the glueballs—have not yet been convincingly detected in experiments. Typical glueball candidates include $f_0(991)$ [1], $\eta(1440)$, $f_2(1720)$ and the G resonances [2]. The interpretation of these resonances would be easier if there were reliable, *ab initio* calculations of the glueball masses. One should not forget that the mechanism that binds gluons into glueballs (i.e. confinement) is non-perturbative and field-theoretic. Consequently, the calculations must be non-perturbative and field-theoretic to be either reliable or *ab initio.*

Many of the theoretical contributions to this conference are based on "QCD-inspired" models. This talk is an update of results from numerical work in lattice gauge theory. Unlike many of the models, lattice QCD *is* QCD. Of course, there are uncertainties associated with the calculations, and a good calculation includes a thorough, honest estimate of the uncertainty.

Almost all of the work reviewed here makes an approximation that is difficult to justify: the quarks are left out. Inasmuch as these calculations use the non-perturbative quantum field theory of gluons, QCD motivates them more than the models. The solution of this pure gauge theory is certainly a necessary first step on the road to solving QCD. Indeed, the first round of lattice calculations with quarks are emerging. They indicate that intermediate mass quarks do not change mass ratios significantly.

The presentation starts with a brief technical review, aimed at non-experts. We shall discuss the framework, paying attention to systematic details. We hope that this approach better prepares the reader for interpreting results in lattice gauge theory realistically. We then present a compilation of results for the string tension and the 0^{++} and 2^{++} glueball masses.

2 Gauge Theory on a Lattice and in a Box

Lattice gauge theory is a generally applicable scheme for non-perturbative calculations in quantum field theory. The ultraviolet divergencies are regulated by replacing (Euclidean) space-time with a discrete lattice. For computer simulations the "space-time" is usually a torus— a finite box with periodic boundary conditions

$$R^4 \rightarrow T^4 \rightarrow N_S^3 \times N_T \text{ lattice.} \tag{1}$$

Writing a for the lattice spacing, $L = N_S a$ is the physical size of the spatial volume, and, as always in the Euclidean formalism, the time extent $T = N_T a$ is related to the physical temperature Θ as follows: $T = 1/\Theta$.

The reason for replacing infinite space-time by a lattice inside a torus is to obtain a finite number of degrees of freedom. We do this not just because the computer demands it. Field theory in an infinite space describes an uncountably infinite number of degrees of freedom. Such a system can be described mathematically as the limit of a sequence of systems with a finite number of degrees of freedom. From this perspective renormalization is no longer a mystery, but a relationship between physically equivalent systems in said sequence. In their numerical work, lattice gauge theorists ask the computer to calculate several physical quantities in a subsequence of these systems. By studying the results, we hope to obtain predictions of, say, glueball masses, in the limits where $a \to 0$ and $L \to \infty$.

On the lattice natural units are set by the lattice spacing a. If M is a mass, in MeV, then the lattice calculation can only determine the product Ma, where a is the lattice spacing. Mass ratios $m_r a/Ma = m_r/M$ are the natural predictions, and one must then choose one mass M ("the mother of all physical quantities") to set the scale before comparing with experiment. One trades in g_0 for M, completely analogously to trading in the bare coupling e_0 of QED for the fine structure constant $\alpha = 1/137$. Furthermore, from the asymptotic freedom of non-Abelian gauge theories, one knows that reducing the bare coupling g_0 reduces the lattice spacing a.

On the torus the rotational symmetry is reduced to cubic symmetry. (The hypercubic lattice breaks the symmetry likewise.) Hence the states are classified by cubic symmetry quantum numbers, rather than spin J. The cubic group has only five irreducible representations, denoted by A_1, A_2, E, T_1 and T_2, with dimensions 1, 1, 2, 3, and 3, respectively. For large enough L (and small enough a) one expects restoration of rotational symmetry, which is signaled by "accidental" degeneracies of cubic multiplets. For example, an E-doublet must combine with a T_2-triplet to form the spin-2-quintet. To emphasize the need to obtain rotational symmetry restoration, we shall use the cubic labels: A_1 for the scalar, and E and T_2 for the two states that ought to form the tensor. On the other hand, the quantum numbers P and C are respected by the torus and the lattice, so they will be denoted by superscripts, in the customary way.

3 Numerical Work

In numerical work one generates an ensemble of configurations $\{U_\mu(x)\}^{(n)}$, $n = 1, \ldots, N_{\mathrm{conf}}$, distributed with weight e^{-S}. (The $U_\mu(x)$ represent the gauge field on a lattice.) The path integral for a correlation function is estimated by

$$C_r(t) = \langle \Phi_r^*(t)\ \Phi_r(0)\rangle \approx \frac{1}{N_{\mathrm{conf}}} \sum_n \Phi_r^*(t; \{U_\mu(x)\}^{(n)})\ \Phi_r(0; \{U_\mu(x)\}^{(n)}) \tag{2}$$

where Φ_r is an interpolating field operator for states with quantum numbers denoted by r. Eq. (2) expresses Monte Carlo integration with importance sampling of the path integral; as $N_{\mathrm{conf}} \to \infty$ the right-hand side converges to the left-hand side. At large enough t the correlation function takes the form

$$C_r(t) = |\langle r, 1|\hat{\Phi}_r|0\rangle|^2 \exp(-m_{1,r}t). \tag{3}$$

Fitting eq. (3) yields the lowest mass $m_{1,r}$ in the channel with r quantum numbers.

Before meaningful numbers can be extracted, one must step through a sequence of lattices, as discussed above. In a rigorous definition of the continuum quantum field theory, one would take limits; the following list denotes them that way and in the "rigorous" order:

1. $N_{\mathrm{conf}} \to \infty$; in this limit the Monte Carlo integration becomes exact. The right-hand side of eq. (2) yields the correct result for the *lattice* theory.

2. $a \to 0$; this is the continuum limit, but it must be approached with $L = N_S a$ and $T = N_T a$ held fixed. Hence, the lattice size parameters N_S and N_T must increase as $1/a$.

3. $N_T a = T \to \infty$, the zero temperature limit; except in work relevant to the QCD phase transition at a non-zero temperature Θ, in which case $T = 1/\Theta$.

4. $N_S a = L \to \infty$, the infinite volume limit.

Treated strictly as limits, all of these steps require an infinite amount of computer time; otherwise they introduce uncertainties. The uncertainty from Limit 1 is *statistical*, because it decreases as $1/\sqrt{N_{\rm conf}}$. The statistical error can also be reduced by variance reduction techniques [3]. The uncertainties of Limits 2-4 are *systematic*. They are best controlled by systematic study of a, T or L dependence, using a sequence of lattices.

A previous review on this subject provides some rules of thumb, indicating when each kind of error has become tolerable [4].

In addition to the non-zero a and finite L systematic effects, there is also some error associated with truncating eq. (3) to one state. One can make this error smaller that the statistical uncertainty by requiring

$$m_{\rm eff}(t) = \ln\left(\frac{C_r(t)}{C_r(t+a)}\right) \tag{4}$$

to be constant in t. If so, one state dominates, and estimates of the mass $m_{1,r}$ and its statistical uncertainty can be obtained from a fit. Incidentally, these fits must take into account that fact that the Monte Carlo estimates for $C_r(t)$ and $C_r(t+a)$ are highly correlated.

4 Results from SU(3)

A nice way to disentangle the various effects is to plot dimensionless, but physical, ratios, say m_r/M. For fixed a one can map out a set of points by varying L; a dimensionless measure on the volume would be $z_M = LM = N_S a M$. Then, by varying a also, one can get a sequence of curves, whose limit gives the continuum limit. The large L value of m_r/M, read off the continuum curve, can be compared to experiments.

Fig. 1 is a plot along these lines, compiling results for glueball masses and the string tension from several groups for the SU(3) gauge group. It is an update of the figure presented before [4, 15], using $z_{\sqrt{K}} = L\sqrt{K}$ as the measure of the volume. The points are from work done in a range of lattice spacing roughly 0.12 fm $\gtrsim a \gtrsim$ 0.05 fm.

The curves are the result of analytical calculations [13]. These calculations start with Lüscher's perturbatively derived effective Hamiltonian for the zero-momentum modes of the gauge field [16], but the spectrum is obtained non-perturbatively [14]. Indeed, for a theorist the left third of fig. 1 is extremely interesting. Two non-trivial non-perturbative approaches to gauge field theory, with completely different approximations, give essentially the same glueball spectrum, in the regime where both are valid.

There are several striking features of the results. First, the ratio $\sqrt{K}/m_{A_1^{++}}$ is surprisingly constant for $z_{\sqrt{K}} > 0.6$. Second, for $0.2 < z_{\sqrt{K}} < 2.0$ the two multiplets that should form the tensor glueball are not at all degenerate. But in the region $1.8 < z_{\sqrt{K}} < 2.8$ the mass of the E representation changes by a factor of two and for $z_{\sqrt{K}} > 3$ the $m_{E^{++}}$ and $m_{T_2^{++}}$ agree within statistical errors. The crossover region is not in a surprising place, $L \approx 1$ fm, but it is intriguing that $m_{E^{++}}$ behaves so differently from $\sqrt{K}$, $m_{A_1^{++}}$ and $m_{T_2^{++}}$.

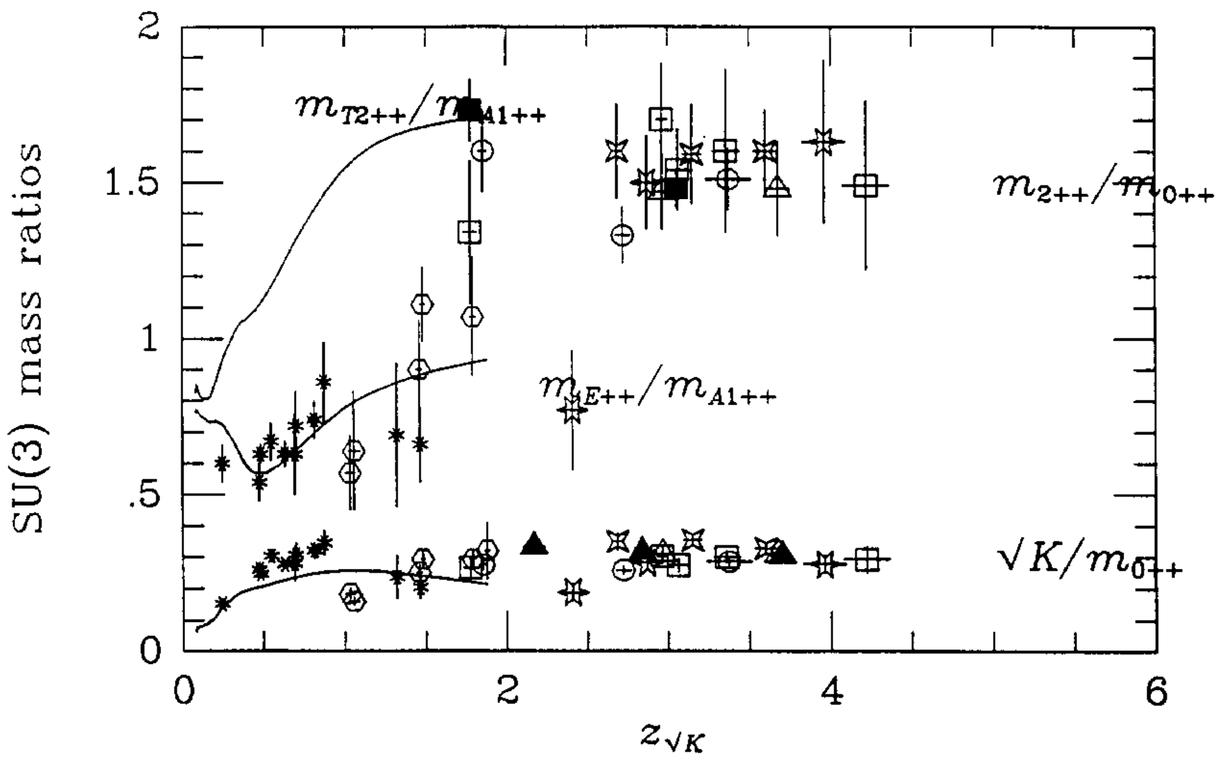

Figure 1: *Plot of dimensionless ratios $\sqrt{K}/m_{A_1^{++}}$, $m_{E^{++}}/m_{A_1^{++}}$ and $m_{T_2^{++}}/m_{A_1^{++}}$ vs. $z_{\sqrt{K}} = L\sqrt{K}$. The symbols are asterisks [5], solid (T_2) and open (K and E) squares [6], solid triangles [7], open triangles [8], four-pointed stars [9], hexagons [10], circles [11], and six-pointed stars [12]. The curves are analytic results [13] valid in small volumes [14].*

Taking $z_{\sqrt{K}} > 2.6$ as close enough to the infinite volume, and averaging, these data yield $\sqrt{K}/m_{0^{++}} = 0.308 \pm 0.020$ and $m_{2^{++}}/m_{0^{++}} = 1.543 \pm 0.082$. The subscripts now refer to spin, because the degenerate E and T_2 masses suggest restoration of rotational symmetry. Setting the scale with $\sqrt{K} = 420$ MeV gives predictions of $m_{0^{++}} = 1370 \pm 90$ MeV and $m_{2^{++}} = 2115 \pm 125$ MeV. These values are tantalizingly close to resonances reported by GAMS [2], but it would be imprudent to draw exciting conclusions.

One series of points, the six-pointed stars [12], are taken from one of the first papers to include quark fields in the calculations. The quark mass is not yet especially small, so it is perhaps not too surprising that the ratios are compatible with pure gauge theory.

5 Concluding Remarks

The overall status in these calculations has not changed much in the past two or three years. Some obvious challenges have been left untouched. First, no one has tried to understand, in detail, the physics in finite volumes, where the $m_{E^{++}}$ changes so much. Second, there are still no results with presentable estimates of the uncertainties for glueball states with exotic J^{PC}. These states cannot mix with $q\bar{q}$ states, and hence the pure gauge predictions ought to be more reliable. Unfortunately, these states seem to lurk under the statistical noise of lattice Monte Carlos, just as they seem to lurk in the backgrounds of experiments.

The next round of lattice calculations of glueball properties will probably be those including intermediate mass quarks. These calculations will encounter the same kinds of challenges as experiments do. The glueballs will not be the lightest states with their quantum numbers; some multi-pion state will be lighter. Unambiguous demonstration of a glueball with normal J^{PC} will require the extraction of two states: the "mostly $q\bar{q}$" and the "mostly gg." Indeed, I question whether the attribute "mostly" will make sense. Since the binding mechanism (i.e. confinement) is a non-perturbative phenomenon, so is the mixing mechanism. Furthermore, the coupling is somewhat strong, so the flavor singlet mesons are likely far from the pure gg or pure $q\bar{q}$ limits. A best description of their structure offered so far is Isgur's "brown muck."

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